# Did the Soviets Collude? A Statistical Analysis of Championship Chess 1940-64

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Abstract

We expand the set of outcomes considered by the tournament literature to include draws and use games from post-war chess tournaments to see whether strategic behavior can be important in such scenarios. In particular, we examine whether players from the former Soviet Union acted as a cartel in international all-play-all tournaments – intentionally drawing against one another in order to focus effort on non-Soviet opponents – to maximize the chance of some Soviet winning. Using data from international qualifying tournaments as well as USSR national tournaments, we consider several tests for collusion. Our results are consistent with Soviet draw-collusion and inconsistent with Soviet competition. Simulations of the period's five premier international competitions (the FIDE Candidates tournaments) suggest that the observed Soviet sweep was a 75%-probability event under collusion but only a 25%-probability event had the Soviet players not colluded.

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Research into how tournament structure affects effort in athletic competition has generated many insights into the problems of optimal compensation in labor and management. In much of this literature, outcomes/output are monotonic functions of effort plus an error term (cf. Ehrenberg and Bognanno, 1990), match-ups end in either wins or losses, and strategic behavior by players is accordingly considered unimportant. Many important economic scenarios, though, are better characterized by the presence of a third outcome that is analogous to a draw, and the existence of such a draw outcome can introduce the potential for strategic behavior. Because of the quality of their data, chess tournaments are ideal for the examination of issues stemming from the possibility of draws. In this paper, we test whether Soviet chess players exploited the existence of the draw and round-robin (i.e., all-play-all) formats, effectively acting as a cartel in international tournaments following World War II.

"Draws" in life are commonplace, and the presence of a draw strategy complicates the problem of encouraging effort. For instance, a player may have the choice of more and less conservative strategies with different chances for a draw. Risk-averse players may pursue less aggressive strategies, trading expected payout for reduced variance.[1] Given that tournament

[1] Consider the following hypothetical. Assume that a large automotive company wishes to motivate one of its divisions to innovate by offering a very high reward in the event of a new design success. But, consistent with much of the tournament literature, the company also proposes no reward or even punishment if the proposed design fails. Under certain assumptions, the division would do its best to produce a terrific design but only if outcome is monotonic in effort. On the other hand, suppose the division had two options: A low-risk design (the draw

chess is not simply stressful but notoriously tiring, we hypothesize that very strong players who wish to collude in round-robin formats can improve their performance against other players by agreeing to early or prearranged draws. Assuming for simplicity that the expected outcome of a game between players of equal strength is half a point each, a peaceful pair of agreed draws produces the same outcome with less effort and risk than taking a win and loss apiece.

There is now a well-established literature on the economics of sports. For our purposes the most relevant work has focused on the importance of tournament structure in motivating effort and in providing incentives for good play (see the overview by Szymanski, 2003). In most cases the applications have been to athletic competitions, whether individual (golf or tennis) or team sports (basketball or soccer). There is also a related literature on the problem of collusion and of cheating in sports which overlaps with this work.[2]

For the purposes of econometric analysis, chess has numerous strengths. First, the outcomes are clear and objective: A win, a draw, or a loss. Moreover, a perfect record of all games is available for virtually all important championship and high level tournament games of

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option) with little chance of being a "home run" but also little chance of being a failure or a high-risk design (the play for the win strategy) with a higher probability of leading to a big success but also a greater chance of serious failure. Depending on the precise conditions, it might actually pay the company to use a less skewed payoff structure if it wants to encourage true innovation. This conjecture and its elaboration are subjects of separate and future research. (For work that treats draws in the context of refinements of the prize structure, see Nalebuff and Stiglitz, 1983).

[2] The most commonly cited work in this genre is Duggan and Levitt's work on sumo wrestling (2002), but there is also a related literature involving biased judging in sporting events (Zitzewitz, 2006; and Garicano, Palacios-Huerta, and Prendergast, 2005).

the modern era. Most important of all is that there exists a rating system which is a precise and accurate reflection of the performances of players and which is an excellent indicator of the relative strengths of players. These ratings are the best unbiased estimates of relative strengths and the differences in ratings correspond to the likelihood that the stronger player will defeat the weaker (cf. Elo, 1968). These Elo-style ratings have since been applied not only to other sports but also to studies of revealed preference rankings in college selection (cf. Avery, et al. 2004).

The question of Soviet collusion during the period when the USSR dominated the struggle for the world championship has been one of the long-standing debates in the chess world. The best-known criticism was aired by Bobby Fischer some weeks after the 1962 Candidates Tournament in Curacao in the magazine *Sports Illustrated* in the article, "The Russians Have Fixed World Chess" (cf. Timman, 2005).[3] In it, he claimed that three of the Soviet players (Paul Keres, Tigran Petrosian, Efim Geller) had agreed to pre-arranged draws amongst themselves in order to conserve energy and to permit themselves to play to the fullest against the non-Soviet competitors. A fourth player (Viktor Korchnoi) was supposedly forced to throw games to the others to minimize the chance of a non-Soviet (probably Fischer) winning the tournament.[4] These allegations have been hard to confirm and many observers have noted that Fischer performed so poorly in that tournament that any collusion was unlikely to have been the cause of his defeat. Nonetheless FIDE, the governing body of the world chess organization, was sufficiently concerned to modify the system for selecting the challenger for the world

[3] See also Plisetsky and Voronkov, 2005.

[4] Korchnoi has claimed that the trio of Geller, Keres, and Petrosian had agreed upon a private arrangement to draw amongst themselves.

championship from 1964 onwards, changing the penultimate Candidates tournament from round-robin to a knockout format.

While Fischer was the most visible of those expressing concerns of draw-collusion, he was not the first. Samuel Reshevsky – the leading US player before the arrival of Bobby Fischer – raised the possibility of non-sporting considerations due to the presence of large numbers of Soviet GMs in the major tournaments of the 1950s.[5] More recently, the Russian grandmaster David Bronstein has suggested that political pressure was put on players during important Candidates tournaments in the 1950s, though observers still disagree about the extent to which outright cheating was involved (cf. Soltis 2002a, Soltis 2002b, and Ree, 2002). Most analyses of the question of Soviet collusion have turned on the evaluation of individual games and the judgment as to whether some were deliberately "lost" or "drawn" in favorable positions.[6] For

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[5] Chess players have also debated the extent to which the Estonian-Soviet grandmaster, Paul Keres, was coerced into losing the world championship to the more politically acceptable Russian, Mikhail Botvinnik (Kingston, 2001).

[6] Timman's book on Curacao (2005) goes into the most detail regarding the games themselves. The testimony of participants since the fall of the Soviet Union makes it more likely than ever that some kind of draw-agreement was in place. Furthermore, Timman's analysis of the critical game between Keres and Petrosian probably provides the strongest "smoking gun" as far as prearranged draws are concerned. The two players decided to a draw in a position that was extremely favorable towards Black. Timman (a world class grandmaster and at one time the top player in the Western world) provided a detailed chess analysis of the position and said, "My conclusion is that Fischer was correct in declaring that Petrosian agreed to a draw in a winning

the most part there has been no systematic analysis of the pattern of wins and draws in the tournaments themselves, though all have noted that there was indeed a large number of draws between the three leading Soviet players in Curacao 1962 (Timman, 2005).

The problem of detecting collusion has naturally received much attention in industrial organization, and our identification strategy has a strong parallel there. Our approach is most in line with the literature that makes comparisons to a broadly acceptable conduct benchmark. For example, Bresnahan (1987) uses the approach to great effect, coupling the benchmark that a firm will set prices "collusively" for all products in its portfolio with the observation that firms may cluster products in some areas of product-space and achieve a local near-monopoly. He then implicitly compares inferred markups of autos that share a portfolio and similar product characteristics with those of autos of similar characteristics but differing ownership. Our identification comes from the observation that a Soviet cartel has no bearing in an all-Soviet tournament. The fact that the leading Soviet players regularly competed with each other in Soviet national tournaments implies that those tournaments can serve as our competitive benchmark as we consider international play. This approach is sufficient for the identification of our predictions regarding the likelihood of draws, but in much of our analysis we supplement it with data on player quality, namely ratings that make use of the entire history of play. The absence of such an objective measure of quality is in part what typically complicates the detection of collusion in industrial organization, and the powerfully predictive ratings in chess suggest that

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position," (Timman, 2005, p. 186). This is especially striking considering that both were tied for first with only 3 rounds to follow.

the question of collusion may be convincingly answered here[7]. In so doing we hope to highlight the potential importance of draws and draw-like scenarios in the tournament literature and the real world.

*The Structure and History of the World Championship1948 to 1962*

Prior to the Second World War, the World Chess Championship was considered the private property of the world champion. He alone could determine which players could challenge him. Championship matches required the solicitation of private funds for the challenger and the champion and the terms of the match were subject to negotiation. If the champion were dethroned, the new champion would have the right to impose conditions on future challengers. Attempts were made to regulate the championships, but for the most part these rules were unenforceable because of the private nature of the title.

The death of the world champion Alexander Alekhine in 1946 allowed the professional organization FIDE to seize control of the world championship and regulate future competitions. They instituted formal regulations for awarding official titles of international master (IM) and international grandmaster (IGM or more commonly GM) and imposed a structure on acquisition of the world title. To replace Alekhine, a special quintuple round robin tournament split between The Hague and Moscow was held with five invited players in 1948. The players were Mikhail Botvinnik, Max Euwe, Keres, Reshevsky, and Vasily Smyslov. Botvinnik emerged as world

[7] Indeed, none of the previous papers on collusion in sports have quality/performance ratings that can rival the modified Elo-Sonas ratings used in this paper. For example, Duggan and Levitt (2002) rely on small-sample win-loss records which take little account of the strengths of all the participants and do not allow fine distinctions both across players and intertemporally.

champion. From that point on, the championships were put on a triennial cycle. The point of the cycle (based upon local competitions arranged by geographic region) was to select players who would play in a qualifying tournament called the Interzonal. The top six players would then join participants other than the champion from the last qualifying cycle (the specific details of qualification and the exceptions therefrom are not important) in a Candidates tournament, which was a multiple (two or four) round-robin event. The winner of this tournament was designated the official Challenger who would then play a 24-game match for the world championship against the incumbent. Should the incumbent be defeated, he was entitled to a rematch. The Candidates tournaments were considered to be difficult and grueling affairs in which the physical stamina of the players could have affected the quality of their play, raising the concern about prearranged draws.

Prearranging draws did not violate any explicit rules, because there was no way to enforce a rule that restricted such behavior. Despite its equal lack of enforceability, deliberately losing a game was more likely to be considered outright fraud. Nonetheless all forms of prearrangement were viewed as unethical or worse. A group of leading players could conceivably increase their chances of success by playing conservatively amongst themselves at first and then if necessary, perhaps even throwing games to whichever member of the group happened to be "in form" as the tournament progressed. In practice, only the USSR had a consistently large enough group of players in the FIDE qualifying (Interzonal and Candidates) tournaments for such a plan to be feasible.

*The Model*

We consider two hypotheses that describe player behavior in chess tournaments. Our null is that individuals choose strategies and exert effort to maximize their own likelihood of winning a particular tournament. We will abstract away from some real-world circumstances and restate this as individuals acting to maximize the probability of winning the game that they are playing.[8] Our alternative hypothesis is that Soviet players act as a cartel in FIDE events, acting within constraints to maximize the probability that some member of the cartel wins the tournament.[9] Soviets in non-FIDE events and non-Soviets in all events, however, act to maximize the probability of winning the game that they are playing.

Within the context of the draw collusion explained above, there are three testable implications of the collusion hypothesis.

1. Games that involve two Soviets and are in FIDE tournaments should be likelier to end in draws than other games.
2. Games that involve two Soviets, are in FIDE events, and end in draws should end with fewer moves than other games that end in draws.
3. Soviet players should outperform their ratings in FIDE tournaments when playing a non-Soviet opponent.

The first prediction is our primary concern, and we approach it in two ways. First, we compare the likelihood of draws in games involving the same Soviets playing particular colors at roughly

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[8] This simplifying assumption runs counter to the observation that a player with a commanding lead going into the last round of a tournament is much likelier to play for a draw.

[9] One plausible such constraint is that, within the cartel, each player wants to win the tournament. Consequently no player is willing to throw matches to other players in the cartel.

the same time across FIDE and URS games. Second, we apply an ordered probit to Soviet and non-Soviet players to consider whether the draw thresholds for Soviet-on-Soviet play in FIDE events systematically differ from those of other events. We address the second prediction using a simple regression of the number of moves using the sample of games that end in draws. We test the third prediction by returning to the ordered probit estimates. To confirm that such outperformance stems from draw collusion, we compare the estimated Soviet outperformance to any outperformance that arose in later knockout tournaments where draw-collusion was impossible. We also explore the nature of the benefits from draw-collusion by examining how variables capturing player fatigue affect outcomes over the course of tournaments.

White's relative advantage over Black will be dependent upon the comparative mastery of the two players. We capture this comparison with the concept of *RatingDif*, White's rating less Black's rating.[10] To explore our third prediction, we would ideally have data on the relative fatigue of the two players, or more plausibly the difference between players' cumulative moves at the start of a game. If draw collusion is important and fatigue-avoidance is a primary mechanism, we expect Soviet players to outperform most visibly toward the end of a tournament and likewise expect that such outperformance will be minimal at the start of a tournament. While we will eventually investigate such fatigue measures, we first consider the concept of *SovietDif* as a cruder measure of this relative advantage. *SovietDif* is the difference of a binary variable denoting whether White is Soviet and the same for Black, taking the values of 1, 0 or -1.

The most obvious issue regarding the threshold against which we compare this relative advantage of White is the inherent advantage of White's position in moving first. We correspondingly do not expect these base thresholds to be symmetric and impose no such

[10] We discuss these ratings in greater detail in our review of the data.

restriction. We further parameterize these thresholds as potentially depending upon whether a game is part of a FIDE tournament (*FIDE*) and whether both players are Soviets and the game is part of a FIDE tournament (*BothSov*FIDE*). The former parameter measures the extent to which all players are likelier to draw (play more conservatively) in FIDE events and the latter parameter captures the collusion hypothesis's first prediction. Preliminary results indicated that a symmetry assumption on these margins was benign, and parameters reflect this symmetry restriction.

Formally, let $RAW_{jkt}$ denote player j's relative advantage as White over player k (Black) at time t. This relative advantage is dependent upon both observed (*RatingDif* and *SovietDif*) and unobserved characteristics: $RAW_{jkt} = X_{jkt}\,\beta + \varepsilon_{jkt}$, where $\varepsilon \sim N(0, \sigma^2)$. Thresholds depend only upon observed characteristics (*FIDE*, *BothSov*FIDE*). Let the winning threshold be denoted $\gamma^{Win}_{jkt} = \gamma^{W} + Z_{jkt}\,\gamma$, so that player j wins as White if $RAW_{jkt} > \gamma^{Win}_{jkt}$. The losing threshold is comparable and denoted as $\gamma^{Lose}_{jkt} = \gamma^{L} - Z_{jkt}\,\gamma$, so that player j loses as White if $RAW_{jkt} < \gamma^{Lose}_{jkt}$. Player j then draws as White if $\gamma^{Lose}_{jkt} < RAW_{jkt} < \gamma^{Win}_{jkt}$.[11]

These parameters are only identified up to scale, and we accordingly normalize the variance of the normal disturbance to $\sigma = 1$. We estimate these parameters using maximum likelihood.

*The Data*

Our data were supplied by Jeff Sonas of Chessmetrics.com and include all non-playoff games of FIDE events and the Soviet (URS) national championship from 1940-64, for a total of

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[11] Ties of RAW and thresholds are assumed to occur with zero probability.

4366 games.[12] For robustness checks, we also included Candidates and World Championship tournaments from 1965-78 and certain additional URS games from 1939-74. We then supplemented this data with information regarding game ordering and ply-count from Chessbase's Mega database 2006. An observation of a game includes the tournament, the players' names, their ratings, which player was White, the ply number (half-moves) at game conclusion, and the outcome. In light of the potential for player movement or defection, whether a player is Soviet reflects the residence of a player at the time of a tournament. We have complete observations for all FIDE games and about 40% of URS games. The most common missing variables are ply-count and the round of the game, and we therefore perform our ply-count regressions and fatigue-based ordered probit on smaller samples.

What makes this data set so valuable for our purposes is the continuity and frequency of the ratings derivation. Ratings in chess that make use of rigorous statistics to produce good estimates of relative player strength are now relatively common, but comparing ratings across different time periods is often complicated by idiosyncratic changes (cf. Elo, 1968 for the pioneering discussion). Sonas uses a weighted and padded simultaneous performance rating formula throughout our sample and updates this rating monthly instead of annually, as is more common (cf. Sonas, 2005, and Sonas, 2006). This specification has been optimized for predictive power for games between world-class players. While the formula is given in Appendix I, there are three primary differences between these Sonas ratings and standard Elo-based ratings. Besides a linear framework that appears to dominate in predictive power the more common

[12] Since most playoffs usually involved a two-person knockout match consisting of several games, the question of draw collusion at this stage is obviously irrelevant.

logistic specification, Sonas ratings weight more recent games more heavily than distant games (looking back four years) and are padded to reward players who play more games.

Retrospective grading allows Sonas to establish ratings that are unbiased estimates of the "true" relative strengths of players. This rating requires a minimum number of observed games to construct, and games without Sonas chess-ratings for both players are dropped from the sample. While this leads to the omission of a few Interzonal games, the vast majority of dropped games are from URS championships. These omissions will presumably increase the average observed skill of Soviet players in URS championships, and thereby make our comparison to FIDE events even more compelling.

Table 1 includes variable names, definitions, and summary statistics of our primary set of observations. Variables that are incorporated into White's relative advantage are at the top. As one would expect from the random way in which White is assigned, both RatingDif and SovietDif have mean zero. From the threshold variables below, we can see that 40% of our full sample's games are from FIDE tournaments and 6% of the total games involve two Soviets playing in a FIDE event. The ply-count variable indicates that the typical draw ended after 36 moves, as a move is composed of two plies (half-moves).

*Results*

We begin our evidence with a comparison of draw lengths in Table 2A. Draws in FIDE events occur after marginally fewer moves than draws in URS championships. More strikingly, draws in FIDE games that involve two Soviets occur substantially sooner than draws in FIDE games involving at least one non-Soviet. This raw data is then consistent with the second prediction of the draw-collusion hypothesis. A similar story appears upon examining draw rates, shown in Table 2B. Consistent with the Soviet collusion hypothesis, draws were likelier in FIDE events than in URS championships (51% vs. 46%). This result, however, is also consistent with the possibility that all players acted more conservatively when they played in FIDE tournaments. We therefore consider the draw rates in FIDE events conditioned on whether both players in a game were Soviet. This result is much more striking: 67% of FIDE games involving two Soviets ended in draws, compared to a 49% draw rate for FIDE games in which at least one player was not Soviet. This, of course, matches the primary prediction of the Soviet collusion hypothesis.

The above comparison, though highly significant, is not quite perfect. Draw rates in URS tournaments may have been depressed as dominant Soviet players take advantage of weaker Soviet opponents and play more aggressive strategies. To address this concern, we perform a comparison that is almost ideal. That is, we compare each FIDE game between two Soviets that ended in a draw to the draw-likelihood of games at around the same time involving the very same two players playing the same colors in all-Soviet tournaments. Because not all Soviets play one another in any given URS tournament, the sample size grows with the window around the FIDE game. Larger windows should lead to more precise estimates but weaken the quality of the comparison, as their relative strengths might have changed significantly over the larger time period.

Table 3 displays the results of this comparison for varying window sizes. As speculated, the URS draw rate in this setting is substantially higher than the overall URS draw rate (60% vs. 46%). Even against this higher draw rate, however, the difference from the FIDE draw rate is always positive. This difference stabilizes around the value of 0.07 when windows are at least +/- four years and becomes statistically significant at conventional levels when windows exceed +/-

six years. Given that the largest sample size is 232, this particular measure seems to offer especially compelling evidence in favor of the hypothesis of draw collusion.

Our next regression builds upon the raw data regarding ply-counts. “Hard-fought” games that end in draws are more likely to last longer than collusive or pre-arranged draws. The latter are more likely to be agreed to at an earlier stage when the position on the board is still not fully resolved and it is not clear than one player should win. At a later stage, the likelihood is much greater that the position will clearly favor one or the other player (to the point where it would be implausible for grandmasters to agree to a draw when one is in a strongly dominant position). Using the sample of games that ended in draws and for which we observe ply-counts (n=2091), we run a least-squares regression of ply-count on the FIDE indicator and an interaction of the FIDE indicator and the Both-Soviet indicator. Those OLS estimates (asymptotic standard errors are in parentheses) are then

$$\underset{}{\text{PlyCt}} = \underset{(1.00)}{75.87} - \underset{(1.63)}{3.22}\text{*FIDE?} - \underset{(2.83)}{12.97}\text{*FIDE?*BothSov?} + e$$

Though the fit is poor, with $R^2 = 0.017$, the results are consistent with our analysis. At least two conclusions can be drawn. One, there is some evidence that draws took marginally more plies in URS championships than in FIDE events. Two, there is strong evidence that draws involving two Soviets in FIDE events took significantly fewer plies than FIDE event draws involving at least one non-Soviet. The cartel’s draws in FIDE events are estimated to occur eight moves sooner than draws in URS championships. In major chess tournaments playing a mere four or five additional moves (10-plies) at a critical stage of the middle game requires considerably more effort, as well as engendering greater risk. The results therefore indicate differences that are of substantive and not simply statistical significance.

Estimates from the ordered probit can be found in Table 4 (I-II). All parameters are precisely estimated. In addition to the obvious results of the difference in ratings being critical, results indicate the presence of a first-mover advantage ($\gamma^W < -\gamma^L$) and generally more conservative play in FIDE events than in URS events. Both predictions of the Soviet collusion hypothesis are also borne out. Soviets did outperform their ratings in FIDE events, and Soviets were likelier to draw against each other in FIDE events than in URS events. The magnitude of these estimates suggest that the average impact of the effort avoided by the Soviets in FIDE events was equivalent to about 68 points in the ratings.

This estimate is strikingly large, and one might reasonably wonder if it is aggregating factors besides the benefit of draw collusion. For example, if Soviet players in international play (but not URS play) receive treatment and resources (such as state-sponsored pre-tournament preparation and training) that are superior to that of other countries' players, attributing the entire outperformance estimate to draw-collusion is an obvious overstatement.[13] To consider the magnitude of this potential overstatement, we examine data from later chess tournaments, specifically the Candidates tournaments and World Championships from 1965 to 1978. These tournaments were played according to a knock-out format rather than the earlier round-robin format. Draw-collusion was therefore impossible, but other sources of Soviet outperformance were still viable.

These tournaments involved 40 two-person matches, 20 of which had a Soviet facing a non-Soviet. Using this set of 40 matches, we compare the actual score of the first player to play White to the score predicted by Table 4(II)'s estimated coefficient of RatingDif and the

[13] The same reasoning would hold for psychological incentives, e.g., fear of punishment for losing in international settings.

winning/losing thresholds. Both scores are scaled by the number of games and thus potential points. Regressing this difference on a constant to capture any systematic errors in our model and the variable SovDif yields the following estimates (standard errors in parentheses):

$$\text{(ActScore – PredScore)/NumGames} = \underset{(0.023)}{0.017} + \underset{(0.033)}{0.012}\ \text{SovDif} + e$$

Soviet outperformance therefore fails to attain statistical significance. Given the small sample size, poor precision is to be expected and economic/substantive significance may be a better standard. Calculations (not shown) indicate that this SovDif coefficient is consistent with Soviets performing at the equivalent of an additional 10 rating points, well below the 68 point outperformance suggested by the earlier ordered probit estimates.[14] This additional evidence then supports the view that Soviet outperformance in international round-robin formats stemmed primarily, and perhaps entirely, from the benefits of draw-collusion.

We continue our estimation by considering the potential mechanisms through which Soviets outperformed against non-Soviets in FIDE events. Suppose that players began a tournament with a plan for effort expended for each game and draw-collusion helped Soviets by enabling them to limit their exertion against fellow Soviets and to concentrate more fully against non-Soviets. Under such a mechanism, we expect to see no systematic difference in the extent of Soviet outperformance at different rounds of a FIDE tournament. If, however, draw-collusion allowed Soviet performance through the simpler mechanism of players being less fatigued as

[14] Imposing the restriction of no intercept makes SovDif's coefficient and the corresponding rating equivalent even smaller.

they advance, then we expect Soviet outperformance to be less early in a tournament and greater at a tournament’s end.

As the necessary round data were scarce for URS events, we re-estimate the ordered probit model using only FIDE games (n=1751) as our new baseline. In addition to dropping the FIDE indicator from the threshold variables, we also include a variety of proxies for the difference of players’ fatigue levels. Table 4 (III-VI) displays results. Estimates shown in (III) replicate our earlier results omitting any FatigueDif variables. Columns (IV-VI) then separately consider the fatigue proxies of prior games played in the tournament (to capture the advantage of bye-rounds), prior plies (in 1000s), and prior games against fellow cartel members.[15] Similar to RatingDif and SovDif, these variables are differenced between players before being incorporated as regressors into the relative advantage of White. We expect the first two variables to have negative coefficients and the third to have a positive coefficient.

While all estimated coefficients have the expected sign, none greatly reduce the log-likelihood. Only the first two variables are significant at the 95% confidence level, and none are significant at the 99% confidence level. Having played one fewer game than one’s opponent is estimated to yield an advantage equivalent to an additional 18 rating points. The estimate on ply-count suggests that having a 100 lower cumulative ply-count (50 moves) offered an advantage of about 8 rating points. Given the arguable importance of these magnitudes, we tentatively conclude that fatigue reduction played at most a complementary role in causing Soviet outperformance. The primary purpose of the Soviet cartel was to allow Soviet players to

[15] Byes play no role in the Soviet collusion story and are included merely to show whether fatigue generally matters in these tournaments.

concentrate their effort more fully upon non-Soviet opponents regardless of a game's temporal location in a tournament.

Table 5 lists some implications of the full-sample, no-fatigue parameter estimates on the probabilities of game outcomes. First consider the baseline of two evenly matched non-Soviets meeting in a FIDE event. The model's estimates indicate that White has a 28% chance of winning, a 54% chance of drawing, and an 18% chance of losing. This compares to the model's implications of expected draw rates in games between two Soviets which mimic the raw draw rates: 48% in URS events and 67% in FIDE events. Assuming Soviet collusion, the probability of an evenly matched Soviet playing White winning against a non-Soviet in a FIDE event rises from 29% to 38%.

*Simulations*

The evidence thus far does not speak to the question of whether this Soviet collusion had important impacts on tournament outcomes. For example, it could be that the Soviets were so dominant that this collusion only marginally increased the likelihood of some Soviet winning a FIDE tournament. To consider this question, we use the original estimates to simulate the five Candidates tournaments that occurred from 1950 to 1962 and determined the challenger to the world champion.

Observed characteristics and the model's estimates imply the probabilities of the three outcomes. Using a draw from the uniform distribution to simulate the outcome of each game, we then re-play each Candidates tournament 10,000 times.[16] The tournament's winner is the player

[16] For example, an observed game's thresholds and relative advantage for White might imply a 10% chance of White losing, a 50% chance of White drawing, and a 40% chance of White

with the most points (1 pt for win, ½ pt for draw, 0 pt for loss).[17] We conclude by considering a competitive counterfactual in which Soviets do not systematically outperform their ratings and they are no more likely to draw against Soviets than against non-Soviets.

Table 6 shows actual and simulated outcomes as well as player characteristics for the double round-robin 1950 Candidates in Belgrade. The limited range of the ratings gives additional perspective regarding the size of the estimated Soviet collusive advantage (68 points). Soviet dominance is also apparent, as seven of the ten participating players are from the USSR. Under the collusive hypothesis, Smyslov was the favorite (winning 25% of the time), but the eventual winner Bronstein and Alexander Kotov were not far behind (each winning 22% of the time). From the Soviet cartel's perspective, a Soviet would have won the Belgrade Candidates 93% of the time. Under the competitive counterfactual, Miguel Najdorf's probability of victory rises from 3% to 13%, and the probability of a Soviet victory falls to 72%.

The 1953 Zurich Candidates (also a double round-robin) is depicted in Table 7. The number of participants increased from ten to fifteen in response to criticisms because of limited Western participation in Belgrade 1950 due to visa and political concerns (the start of the Cold War). Hence the successful players from the regular cycle were augmented by the strongest Western players that were felt by FIDE to have been arbitrarily cut off from participating in the

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winning. Letting e be the uniform draw, the simulated game ends in a loss if $e<0.1$, a win if $e>0.6$, and a draw otherwise.

[17] Two-way ties are resolved with the best of twelve format used in the 1950 Candidates between Isaac Boleslavsky and Bronstein, and ties involving more than two players are resolved using a double round-robin format. Ties at the end of these tie-breakers are resolved using a random number generator.

previous cycle. By all accounts, Najdorf and Reshevsky were the strongest active players in the West and Euwe of Holland – though now past his prime – was the only living ex-world champion. Retroactive grading has shown that Reshevsky was the favorite going into the Zurich 1953 Candidates Tournament.

Of all our simulations, Zurich shows the greatest difference between the probable winners when going from independent play to collusion. Though legendary for the quality of the games played and the high quality of the top competitors, continued concerns about Soviet collusion and political pressure against the Estonian Soviet Keres have dogged the reputation of this tournament. Our calculations now suggest that the real loser from any collusion was likely to have been the American GM Reshevsky – the strongest American player in the decades prior to the rise of Bobby Fischer, and arguably the second strongest American player in the twentieth century. Our calculations indicate that he had a 27% chance of winning a fair tournament. With collusion, his chances fell to 8%. After the loss in Zurich, Reshevsky continued to play at a high level but never had a realistic chance of challenging for the world championship again.

Amsterdam 1956 (Table 8) is less interesting because the only non-Soviet in the top group was Oscar Panno who was never among the best half dozen in the world. However, it does illustrate the obvious point that Soviet dominance was not entirely a result of collusive behavior. Most of the world's top players came from the USSR. Simulations indicate that even under our competitive counterfactual, there was a 96% chance of a Soviet advancing in that tournament.

Yugoslavia 1959 (Table 9) saw the debut in the Candidates tournaments of Bobby Fischer, but he was only 16 years old and nowhere near his peak strength. Conversely, the young genius Mikhail Tal from Latvia had enjoyed a meteoric rise to become the leading player in the USSR and then the world. He was a rightful favorite. Moreover, in the case of young players with rapidly increasing ability, it is well-established that ratings have a tendency to underestimate their strength, which changes quickly as they approach their peak. Tal won the tournament convincingly and then went on to become world champion by defeating Botvinnik. However, poor health contributed to his losing the title back to Botvinnik in 1961. He never fully overcame his health problems, though he remained one of the chess elite for two more decades.

Curacao 1962 (Table 10) was the famous tournament in which a mature Fischer accused the Soviets of colluding – an accusation supported by Korchnoi after his defection. However, Fischer's ratings suggest that he was not the *ex ante* favorite, even if his rating/performances put him a few points shy of the leaders. Even if his true rating were higher (because Fischer had been improving rapidly in the year or two previous to the tournament), Fischer's poor form/performance in the early rounds of Curacao meant that he had no realistic chance of winning. The Monte Carlo simulations do suggest, however, that he would have suffered from any collusion since his no-collusion probability of winning was 19%, which dropped to 3% with collusion. Note that we assumed that all the Soviets were colluding and did not model the specific claim of Korchnoi that collusion was a private arrangement of Keres, Petrosian, and Geller.

*Conclusion*

No one can know with certainty what sorts of pressures were imposed on Soviet players during the heyday of the Soviet Union – especially in as politically significant a sport as chess.[18] Some such as Korchnoi or Yuri Averbakh – both top Soviet GMs – have testified to the political pressure faced by players to kowtow to the regime, though others have questioned the accuracy of these testimonies or the extent to which games were "fixed" (e.g. Kingston, 2002). Many of the leading players of that period have now passed away and many important state documents from the era have not yet been declassified. However, it is not hard to see how the elevation of championship chess to a sport that exemplified the superiority of Homo Sovieticus would support both formal and informal pressure on players to bend the rules. Even in a non-ideological environment, it is possible that players from the same nation might have been tempted to come to collusive arrangements in important tournaments. Given the highly delicate nature of Soviet relations with the West in the later years of Stalin's rule and in the decade following his death, it should not be surprising that adherence to fair play was not of the utmost importance. Furthermore, a top Soviet player was in a unique position. He was able to travel abroad – a highly restricted and desirable privilege – and he could earn foreign currency should he succeed in foreign tournaments. Moreover, leading players were provided with generous

[18] Soviet emigré and international grandmaster Genna Sossonko tells of a story, that "at one meeting of the Chess Federation, when the behavior of a player who had committed some misdemeanor was being discussed, [former military prosecutor and head of the chess section of the USSR] Baturinsky said heatedly: 'During the war we used to shoot such people'" (Sosonko, 2003, p. 101).

support from the state and enjoyed numerous other privileges not enjoyed by lesser players or by the general run of Soviet professionals.[19]

Thus, the finding that the Soviets were more likely to draw amongst themselves in critical FIDE tournaments than they were when playing other masters should not come as a surprise. But our paper has been the first to provide strong statistical evidence in support of this result. More important, we have shown that such collusion clearly benefited the Soviet players and led to performances against the competition in critical tournaments that were noticeably better than would have been predicted on the basis of past performances and on their relative ratings. The likelihood that a Soviet player would have won every single Candidates tournament up to 1963 was less than one out of four under an assumption of no collusion, but was higher than three out of four when the possibility of draw collusion is factored in.

As many chess writers have observed, Fischer was not a strong enough favorite to be severely harmed by the draw collusion in the notorious Candidates Tournament in Curacao 1962. Nonetheless, all non-Soviet players suffered from the effects of collusion. More significant is that the American GM Sammy Reshevsky seems to have been the greatest victim of collusive behavior. He was the *ex ante* favorite in the Zurich 1953 Candidates' Tournament and his second place finish in the tournament was strong enough that even small collusive effects might

[19] Even as late as 1971 – long after the era of Stalin – performance in chess was treated with utmost seriousness. After losing to Fischer 6-0 in 1971, GM Mark Taimanov lost his title of "Merited Master of Sports," was kicked off the Soviet national team, was forbidden to play in international tournaments for two years, and was even forbidden from performing his secondary occupation as a professional pianist (Plisetsky and Voronkov, 2005, p. 237).

have meant the difference between success and failure. By all accounts, Zurich was Reshevsky's last and perhaps best chance to play a match for the World Chess Championship.

In addition to the relevance of our results for the history of chess, the addition of draws to the possible outcomes of a contest suggests interesting modifications to the standard tournament literature. Almost all of the existing research work in the economics literature has focused on competitions where the scores are assumed to be monotonic in effort (e.g., golf) or where the outcomes of a match are either a win or a loss (e.g., tennis or most team sports). The possibility of a draw outcome and the fact that the choice of strategy might not affect the expected value of a match but might affect the distribution of outcomes (more draws with fewer wins and losses vs. a higher chance of a win with a higher risk of a loss) pose interesting problems from a theoretical and empirical standpoint (cf. Hvide, 1999). Future research can be devoted to exploring the theoretical implications of this modification to the standard model. One can also ask how the possibility of strategies that "keep the draw in hand" might affect the applications of the tournament literature to structuring incentives in strategic competitions or in designing incentive pay packages in managerial hierarchies. Finally, we draw attention to the underexploited data from chess tournaments. The sheer number of games that have been played and recorded (running into several million games) and the quality of the chess ratings system in place suggest the possibility of valuable microeconometric studies of the economics of sports, tournament models, and the effects of rules changes on performance.

**References**

Avery, Christopher, Mark Glickman, Caroline Hoxby, and Andrew Metrick (2004). "A Revealed Preference Ranking of U.S. Colleges and Universities." Working paper.

Bresnahan, Timothy (1987). "Competition and Collusion in the American Automobile Market: The 1955 Price War." *Journal of Industrial Economics*, vol. 35, no. 4, pp. 457-482.

Chessbase, 2006. Mega database 2006. Chessbase, GMBH.

Duggan, Mark and Steven Levitt (2002). "Winning Isn't Everything: Corruption in Sumo Wrestling," *American Economic Review*, vol. 92, no. 5, pp.1594-1605.

Ehrenberg, Ronald and Michael L. Bognanno (1990). "Do Tournaments Have Incentive Effects?" *Journal of Political Economy,* vol. 98, no. 6, pp. 1307-1324.

Elo, Arpad (1968). *The Rating of Chessplayers Past and Present*. Arco.

Garicano, Luis, Ignacio Palacios-Huerta, and Canice Prendergast (2005). "Favoritism Under Social Pressure." *Review of Economics and Statistics*, 2, May, 208-216.

Hvide, Hans (1999). "Tournament Rewards and Risk-Taking," working paper. Norwegian School of Economics.

Kingston, Taylor (2001). "The Keres-Botvinnik Case Revisited: A Further Survey of the Evidence," Chesscafe.com. *The Skittles Room*. 10/08/2001.

Kingston, Taylor (2002). "Yuri Averbakh: An Interview with History, Part I," Chesscafe.com. *The Skittles Room*. 06/121/2002.

Nalebuff, Barry and Joseph Stiglitz (1983). "Prizes and Incentives: Towards a General Theory of Compensation and Competition," *Bell Journal of Economics*, vol. 14, no.1, pp. 21-43.

Plisetsky, Dmitry and Sergey Voronkov (2005). The Russians Versus Fischer. Second Edition. (Everyman: London).

Ree, Hans (2002). "Footnotes to History." Chesscafe.com. *Dutch Treat*. 03/25/2002.

Soltis, Andrew (2002a). "Treachery in Zurich: Part One," Chesscafe.com. *The Skittles Room*. 01/29/2002.

Soltis, Andrew (2002b). "Treachery in Zurich: Part Two," Chesscafe.com. *The Skittles Room*. 02/24/2002.

Sonas, Jeff (2005). "The Greatest Chess Player of All Time – Part I." Chessbase News. April 24, 2005. http://www.chessbase.com/newsdetail.asp?newsid=2345

Sonas, Jeff (2006). "Historical Ratings." Chessmetrics.com. http://db.chessmetrics.com/CM2/HistoricalRatings.asp?Params=.

Sosonko, Genna (2003). *The Reliable Past*. (New In Chess: The Netherlands).

Szymanski, Steven (2003). "The Economic Design of Sporting Contests." *Journal of Economic Literature*. 41, pp. 1137-1187.

Timman, Jan (2005). *Curacao 1962*. (New In Chess: The Netherlands).

Zitzewitz, Eric (2006) "Nationalism in Winter Sports Judging and its Lessons for Organizational Decision Making," *Journal of Economics and Management Science*. Spring, pp. 66-99.

**Table 1: Variables**

| | Definition | obs | mean | std |
|---|---|---|---|---|
| PlyCountDraws | Half-moves in games that end in draws | 2091 | 73.33 | 34.67 |
| RatingDif | (White Rating - Black Rating)/100 | 4366 | 0.00 | 1.00 |
| SovietDif | WhiteSov - BlackSov | 4366 | 0.00 | 0.42 |
| FIDE | 1 if FIDE event, 0 o/w | 4366 | 0.40 | |
| BothSov | 1 if both players Soviet, 0 o/w | 4366 | 0.66 | |
| FIDE*BothSov | interaction | 4366 | 0.06 | |

**Table 2: Comparison of means**

A: Ply-Counts of Draws (n=2091)

| | n | mean |
|---|---|---|
| FIDE | 900 | 69.96 |
| URS | 1191 | 75.87 |
| FIDE\|BothSov=0 | 714 | 72.64 |
| FIDE\|BothSov=1 | 186 | 59.68 |

B: Draw Rates (n=4366)

| | n | mean |
|---|---|---|
| FIDE | 1751 | 0.514 |
| URS | 2615 | 0.460 |
| FIDE\|BothSov=0 | 1472 | 0.485 |
| FIDE\|BothSov=1 | 279 | 0.667 |

**Table 3: Almost Ideal Draw Rate Comparison**

* comparison of draw rates in FIDE games vs matching URS games
* same players, same colors, varying size of window of URS games
* bigger windows increase sample and precision, lessen appropriateness of comparison

| Window, N | URS DR | FIDE + | Window, N | URS DR | FIDE + |
|---|---|---|---|---|---|
| w=+/-1 | 0.612 | 0.052 | w=+/-6 | 0.604 | 0.069 |
| n = 67 | (0.057) | (0.081) | n=226 | (0.028) | (0.039) |
| | | | | | |
| w=+/-2 | 0.608 | 0.028 | w=+/-7 | 0.599 | 0.076 |
| n=136 | (0.040) | (0.056) | n=228 | (0.027) | (0.038) |
| | | | | | |
| w=+/-3 | 0.609 | 0.040 | w=+/-8 | 0.593 | 0.080 |
| n=168 | (0.035) | (0.049) | n=229 | (0.027) | (0.038) |
| | | | | | |
| w=+/-4 | 0.594 | 0.064 | w=+/-9 | 0.591 | 0.079 |
| n=205 | (0.031) | (0.044) | n=230 | (0.026) | (0.037) |
| | | | | | |
| w=+/-5 | 0.601 | 0.067 | w=+/-10 | 0.587 | 0.083 |
| n=220 | (0.029) | (0.041) | n=232 | (0.026) | (0.037) |

Estimated standard errors in parentheses

**Table 4: Ordered Probit (standard errors beneath point estimates)**

| | I | II | III | IV | V | VI |
|---|---|---|---|---|---|---|
| RatingDif | 0.40 | 0.37 | 0.45 | 0.45 | 0.44 | 0.45 |
| | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 | 0.03 |
| SovDif | --- | 0.25 | 0.21 | 0.20 | 0.19 | 0.19 |
| | | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 |
| FatigueDif | | | | | | |
| Games | --- | --- | --- | -0.082 | --- | --- |
| | | | | 0.039 | | |
| Ply/1000 | --- | --- | --- | --- | -0.346 | --- |
| | | | | | 0.151 | |
| BothSov | --- | --- | --- | --- | --- | 0.008 |
| | | | | | | 0.009 |
| Thresholds | | | | | | |
| Win: $\gamma^W$ | 0.52 | 0.48 | 0.59 | 0.59 | 0.59 | 0.58 |
| | 0.02 | 0.02 | 0.04 | 0.04 | 0.04 | 0.04 |
| Lose: $\gamma^L$ | -0.89 | -0.84 | -0.93 | -0.93 | -0.93 | -0.93 |
| | 0.02 | 0.02 | 0.04 | 0.04 | 0.04 | 0.04 |
| FIDE? | --- | 0.09 | | | | |
| | | 0.03 | | | | |
| FIDE*BothSov? | --- | 0.24 | 0.24 | 0.23 | 0.24 | 0.24 |
| | | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| log-lik | 4286.90 | 4251.31 | 1604.32 | 1601.93 | 1601.81 | 1603.95 |
| n | 4366 | 4366 | 1751 | 1751 | 1751 | 1751 |

**Table 5: Implications of estimates**

A. Two evenly matched non-Soviets in FIDE event

| | |
|---|---|
| Pr(White wins) | 0.2860 |
| Pr(White draws) | 0.5378 |
| Pr(White loses) | 0.1762 |

B. Two evenly matched Soviets in FIDE event

| | |
|---|---|
| Pr(White wins) | 0.2097 |
| Pr(White draws) | 0.6697 |
| Pr(White loses) | 0.1205 |

C. Evenly matched Soviet and non-Soviet (Soviet playing White)

| | |
|---|---|
| Pr(White wins) | 0.3774 |
| Pr(White draws) | 0.5041 |
| Pr(White loses) | 0.1184 |

**Table 6: Monte Carlo of Budapest 1950 Candidates**

* 10,000 simulations using baseline ordered probit estimates (Table 3-II)
* 10-player, double round robin (play each opponent 1x as White and 1x as Black)

| | | | | **Cartel** | | **No Cartel** | |
|---|---|---|---|---|---|---|---|
| Players | Rating | Sov? | ActScore | E(PredScore) | % win | E(PredScore) | % win |
| Boleslavsky | 2722.17 | Y | 12 | 9.45 | 0.099 | 9.02 | 0.074 |
| Bronstein | 2753.67 | Y | 12 | 10.08 | 0.224 | 9.73 | 0.177 |
| Flohr | 2659.34 | Y | 7 | 8.25 | 0.014 | 7.63 | 0.009 |
| Keres | 2721.96 | Y | 9.5 | 9.45 | 0.098 | 9.00 | 0.074 |
| Kotov | 2752.53 | Y | 8.5 | 10.06 | 0.224 | 9.70 | 0.175 |
| Lilienthal | 2660.19 | Y | 7 | 8.25 | 0.018 | 7.64 | 0.011 |
| Najdorf | 2741.06 | N | 9 | 8.38 | 0.035 | 9.44 | 0.126 |
| Smyslov | 2760.36 | Y | 10 | 10.19 | 0.252 | 9.85 | 0.203 |
| Stahlberg | 2731.50 | N | 8 | 8.17 | 0.025 | 9.22 | 0.098 |
| Szabo | 2709.60 | N | 7 | 7.72 | 0.012 | 8.77 | 0.055 |
| | | | | Soviet win? | 0.929 | | 0.721 |

Notes: 2251 ties are predicted under collusion (79% 2-way, 18% 3-way, 3% 4-way or more),
2129 ties are predicted under no-cartel (80% 2-way, 17% 3-way, 3% 4-way or more)

**Table 7: Monte Carlo of Zurich 1953 Candidates**

* 10,000 simulations using baseline ordered probit estimates (Table 3-II)
* 15-player, double round robin (play each opponent 1x as White and 1x as Black)

| | | | | **Cartel** | | **No Cartel** | |
|---|---|---|---|---|---|---|---|
| Players | Rating | Sov? | ActScore | E(PredScore) | % win | E(PredScore) | % win |
| Averbakh | 2645.01 | Y | 13.5 | 12.60 | 0.004 | 11.44 | 0.001 |
| Boleslavsky | 2722.33 | Y | 13.5 | 14.88 | 0.082 | 13.99 | 0.045 |
| Bronstein | 2723.87 | Y | 16 | 14.95 | 0.091 | 14.06 | 0.047 |
| Euwe | 2687.61 | N | 11.5 | 11.52 | 0.001 | 12.85 | 0.011 |
| Geller | 2734.03 | Y | 14.5 | 15.23 | 0.118 | 14.39 | 0.063 |
| Gligoric | 2707.39 | N | 12.5 | 12.12 | 0.003 | 13.46 | 0.021 |
| Keres | 2721.02 | Y | 16 | 14.88 | 0.085 | 14.00 | 0.047 |
| Kotov | 2727.00 | Y | 14 | 15.01 | 0.092 | 14.14 | 0.053 |
| Najdorf | 2753.04 | N | 14.5 | 13.69 | 0.029 | 15.04 | 0.128 |
| Petrosian | 2717.44 | Y | 15 | 14.72 | 0.066 | 13.82 | 0.038 |
| Reshevsky | 2780.99 | N | 16 | 14.58 | 0.084 | 15.92 | 0.269 |
| Smyslov | 2764.92 | Y | 18 | 16.12 | 0.272 | 15.40 | 0.175 |
| Ståhlberg | 2721.93 | N | 8 | 12.65 | 0.007 | 14.00 | 0.042 |
| Szabó | 2716.21 | N | 13 | 12.45 | 0.006 | 13.79 | 0.032 |
| Taimanov | 2713.92 | Y | 14 | 14.62 | 0.061 | 13.70 | 0.030 |
| | | | | Soviet win? | 0.870 | | 0.498 |

Notes: 2021 ties are predicted under collusion (80% 2-way, 16% 3-way, 4% 4-way or more),
1957 ties are predicted under no-cartel (79% 2-way, 17% 3-way, 4% 4-way or more)

**Table 8: Monte Carlo of Amsterdam 1956 Candidates**
* 10,000 simulations using baseline ordered probit estimates (Table 3-II)
* 10-player, double round robin (play each opponent 1x as White and 1x as Black)

| | | | | **Cartel** | | **No Cartel** | |
|---|---|---|---|---|---|---|---|
| Players | Rating | Sov? | ActScore | E(PredScore) | % win | E(PredScore) | % win |
| Bronstein | 2769.63 | Y | 9.5 | 10.63 | 0.192 | 10.10 | 0.180 |
| Filip | 2652.84 | N | 8 | 6.70 | 0.001 | 7.55 | 0.004 |
| Geller | 2719.95 | Y | 9.5 | 9.68 | 0.050 | 9.01 | 0.047 |
| Keres | 2782.23 | Y | 10 | 10.87 | 0.248 | 10.37 | 0.245 |
| Panno | 2672.63 | N | 8 | 7.10 | 0.002 | 7.97 | 0.011 |
| Petrosian | 2755.26 | Y | 9.5 | 10.37 | 0.145 | 9.79 | 0.135 |
| Pilnik | 2639.45 | N | 5 | 6.42 | 0.000 | 7.25 | 0.003 |
| Smyslov | 2793.39 | Y | 11.5 | 11.07 | 0.314 | 10.61 | 0.308 |
| Spassky | 2714.37 | Y | 9.5 | 9.56 | 0.046 | 8.86 | 0.043 |
| Szabó | 2695.05 | N | 9.5 | 7.61 | 0.003 | 8.49 | 0.023 |
| | | | | Soviet win? | 0.995 | | 0.959 |

Notes: 1936 ties are predicted under collusion (83% 2-way, 15% 3-way, 2% 4-way or more),
1882 ties are predicted under no-cartel (82% 2-way, 15% 3-way, 3% 4-way or more)

**Table 9: Monte Carlo of Yugoslavia 1959 Candidates**
* 10,000 simulations using baseline ordered probit estimates (Table 3-II)
* 8-player, quadruple round robin (play each opponent 2x as White and 2x as Black)

| | | | | **Cartel** | | **No Cartel** | |
|---|---|---|---|---|---|---|---|
| Players | Rating | Sov? | ActScore | E(PredScore) | % win | E(PredScore) | % win |
| Benko | 2649.69 | N | 8 | 10.30 | 0.000 | 11.44 | 0.004 |
| Fischer | 2699.83 | N | 12.5 | 12.04 | 0.003 | 13.22 | 0.031 |
| Gligoric | 2736.03 | N | 12.5 | 13.29 | 0.014 | 14.50 | 0.109 |
| Keres | 2742.92 | Y | 18.5 | 15.95 | 0.168 | 14.72 | 0.143 |
| Olafsson | 2669.53 | N | 10 | 10.97 | 0.000 | 12.13 | 0.008 |
| Petrosian | 2753.12 | Y | 15.5 | 16.26 | 0.224 | 15.09 | 0.189 |
| Smyslov | 2761.64 | Y | 15 | 16.53 | 0.284 | 15.38 | 0.243 |
| Tal | 2765.75 | Y | 20 | 16.66 | 0.307 | 15.53 | 0.272 |
| | | | | Soviet win? | 0.983 | | 0.848 |

Notes: 1428 ties are predicted under collusion (89% 2-way, 10% 3-way, 1% 4-way or more),
1533 ties are predicted under no-cartel (87% 2-way, 12% 3-way, 1% 4-way or more)

**Table 10: Monte Carlo of Curacao 1962 Candidates**

* 10,000 simulations using baseline ordered probit estimates (Table 3-II)
* 8-player, quadruple round robin (play each opponent 2x as White and 2x as Black)

| | | | | **Cartel** | | **No Cartel** | |
|---|---|---|---|---|---|---|---|
| Players | Rating | Sov? | ActScore | E(PredScore) | % win | E(PredScore) | % win |
| Benko | 2619.689 | N | 12 | 8.48 | 0.000 | 9.80 | 0.001 |
| Filip | 2672.711 | N | 7 | 10.14 | 0.001 | 11.52 | 0.007 |
| Fischer | 2760.832 | N | 14 | 13.05 | 0.029 | 14.48 | 0.189 |
| Geller | 2740.699 | Y | 17 | 14.82 | 0.136 | 13.88 | 0.110 |
| Keres | 2742.707 | Y | 17 | 14.82 | 0.136 | 13.86 | 0.105 |
| Korchnoi | 2769.476 | Y | 13.5 | 15.59 | 0.287 | 14.75 | 0.235 |
| Petrosian | 2782.001 | Y | 17.5 | 16.05 | 0.409 | 15.29 | 0.349 |
| Tal** | 2768.898 | Y | 7 | 12.04 | 0.002 | 11.41 | 0.004 |
| | | | | Soviet win? | 0.970 | | 0.803 |

Notes: 1455 ties are predicted under collusion (89% 2-way, 10% 3-way, 1% 4-way or more),
1410 ties are predicted under no-cartel (87% 2-way, 12% 3-way, 1% 4-way or more)

** Tal withdrew for health reasons after 21 games (i.e., he missed 7 games)

Appendix I: Sonas Ratings

Sonas' Chessmetric ratings are constructed monthly using a weighted and padded simultaneous technique. Both the weighting of recent games over distant games and the padding to reward more frequent games differ from the current FIDE ratings so that ratings are not comparable. Performance ratings (PR), which apply to a single event, are calculated using the formula

$$PR = AOR + (PS - 0.5)*850$$

where AOR denotes a game-weighted average of opponents' ratings and PS denotes the percentage of potential points scored. Using all of a month's events to calculate PR and letting NG denote the number of games played during a month, a single month's rating takes the form

$$MonthRating = \frac{(PR * NG) + (AOR * 4) + (2300 * 3)}{(NG + 7)} + 43$$

The padding takes the form of a reversion to the scores of one's opponents and to the score of 2300, though this reversion will be less notable for those players who compete frequently. The full Sonas rating is then a weighted average of the month ratings over the past four years:

$$SonasRating_m = \left(\frac{1}{24.5}\right)\sum_{t=0}^{47}\left(1 - \frac{t}{48}\right)MonthRating_{m-t}$$

Like the original Elo ratings and current FIDE ratings, the original ratings come from a simultaneous performance rating calculation. Using a closed pool of players, iteratively applying the Sonas rating formula will converge to the "true" Sonas ratings, regardless of the initial guess of ratings. Other players' ratings are then calculated based upon their interactions with the original set of players, and so forth. Sonas begins his calculations with the year 1840 and, for each four year period, uses a top 10 player who was highly active in the prior four years as the initial seed. The closed pool is effectively eleven degrees of separation from this seed, a large number that was necessary for calculating ratings during earlier years. Sonas ratings are then standardized so that the average score of players ranked #3 through #20 are the same across months. Only players who competed in at least 5 games in a month are considered. (http://db.chessmetrics.com/CM2/Formulas.asp)

# Appendix II

## Official World Chess Champions and Their Challengers 1948-1990

| Year [winners in bold] | Champion | Challenger | |
|---|---|---|---|
| 1948 | **Botvinnik** | Initial championship tournament | |
| 1951 | **Botvinnik** | Bronstein | [tie] |
| 1954 | **Botvinnik** | Smyslov | [tie] |
| 1957 | Botvinnik | **Smyslov** | |
| 1958 | Smyslov | **Botvinnik** | |
| 1960 | Botvinnik | **Tal** | |
| 1961 | Tal | **Botvinnik** | |
| 1963 | Botvinnik | **Petrosian** | |
| 1966 | **Petrosian** | Spassky | |
| 1969 | Petrosian | **Spassky** | |
| 1972 | Spassky | **Fischer** | |
| 1975 | Fischer | **Karpov** | [default] |
| 1978 | **Karpov** | Korchnoi | |
| 1981 | **Karpov** | Korchnoi | |
| 1984/85 | Karpov | Kasparov | unfinished |
| 1985 | Karpov | **Kasparov** | |
| 1986 | **Kasparov** | Karpov | |
| 1987 | **Kasparov** | Karpov | [tie] |
| 1990 | **Kasparov** | Karpov | |

Except for 1948 all championships were 2 player matches of many games.
All Soviet players except Fischer (USA).
Korchnoi defected in 1976 and eventually came to represent Switzerland but not in 1978.
In the event of a tie, the champion retained the title.
Until 1963, the champion had the right to an automatic rematch if he lost.  But there was no rematch rule 1963-1975.
The rematch clause was reinstated with Karpov.
Fischer declined to defend his title in 1975 over a dispute with FIDE.
The 1984/85 Karpov Kasparov match was broken off and a new match played in 1985.